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Abstract. When agents have opportunities in both selling and non-selling related activities, the rising opportunity cost of time induces greater focus on the former, with contracted support to pursue unrelated real estate activities. This drives a positive relationship between specialization and income. Nonetheless, income may decline empirically with greater specialization when the population of most specialized agents includes those with either low opportunity costs or the least ability to earn ancillary income. Data drawn from a multi-year survey of real estate professionals indicate that income rises with greater specialization except for the most specialized. The latter result is consistent with the notion that mix of agents at the highest specialization levels enjoy different opportunity costs or ranges of income opportunities.

Keywords: agent income, agent sales effort, agent specialization

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1. Introduction

Real estate agents and brokers are an integral, yet imperfectly understood, part of the housing market. The existing theoretical and empirical real estate brokerage literatures focus solely on brokers' primary activities that are directly related to obtaining listings and completing sales. While a reasonable place to begin, it seems that deeper understanding requires a broader view of brokers and agents that includes opportunities for non-selling related real estate activities--like consulting, appraising, or management--that may detract from or reinforce sales income potential. In order to begin filling this gap in the literature, this paper develops a model of broker effort allocation among selling and non-selling related activities and offers empirical evidence relating effort allocation to individual agent performance.

The real estate brokerage literature can be organized into distinct branches following two different analytical approaches. One approach studies how brokers and agents influence market outcomes, typically measured in terms of selling price and/or liquidity.[1] Most of this work focuses on firm level effects on house prices, but there is a modest body of empirical evidence pertaining to individual agent effects on prices or marketing time (Jud and Winkler 1994, Jud et al. 1996, Levitt and Syverson 2008, Munneke and Yavas 2001, Rutherford et al. 2005, Yang and Yavas 1995, Zumpano, et al. 1996). The second approach focuses on individual agent performance measured in terms of income rather than effects on selling price or marketing time.[2] These studies examine the usual factors suggested by human capital theory, including agent

[1] See Benjamin et al. (2000), Turnbull and Dombrow (2007), and Yavas (1994) for reviews of the relevant literature.

[2] These studies include Allen et al. (2003), Crellin et al. (1988), Follain et al. (1987), Glower and Hendershott (1988), Johnson et al. (2007), Jud and Winkler (1998), and Sirmans and Swicegood (1997). See Benjamin et al. (2000) for a comprehensive summary of results.

experience and training, firm characteristics, and personal characteristics like agent age, race, and sex. The more recent efforts introduce aspects of agent behavior unique to the brokerage environment, like concentrating on listing or selling (Johnson et al. 2007) and willingness to adopt new technology (Benjamin et al. 2002). Our focus on agents' effort mixes across selling related and non-selling related real estate activities broadens the scope of this agent income literature.

This paper is organized as follows. Section 2 offers a stylized model of individual broker effort allocation among various real estate activities. Section 3 discusses the survey data used in the empirical study. Section 4 reports and interprets the empirical evidence in light of the theoretical model. Section 5 concludes.

2. Agent Effort Allocation

Consider a single agent who must choose how much effort to expend on primary selling activities and how much effort to expend on ancillary income-producing activities, such as consulting, property management, or appraising. While sales activities require own effort, the agent, however, can choose to hire someone else to engage in non-sales activities. Denote effort expended on showing houses and other direct selling activities as S and the total effort expended on other non-selling real estate activities as N.

Expected commission income from selling or listing properties is the sum of the probability of sale times the selling price, that is, the expected number of units sold times the expected selling price per unit. Since the probability of selling any given unit and the selling price of that unit both are functions of the total time or effort expended by the agent and assistants in the sales process, the expected commission income of the agent is also a function of these efforts. In addition, though, the agent can spend time or expend effort engaging in ancillary income producing activities, which include general marketing, consulting, property management, etc. Some of these activities produce income that is not directly related to commission income, but even non-selling activities like consulting or property management may have positive feedback effects on the flow of commission income in the long run. Given the inability to completely segment activities into those that solely influence selling income and those that solely influence non-sales income, we simply focus on the agent's total expected income from all real estate activities over the relevant production period, assuming that expected income, I, is a function of own selling effort and total other or non-selling related effort,

$$\begin{aligned} I &= \theta f(S, N) \\ &= \theta f(S, N_o + N_a) \end{aligned}$$

where S is selling effort provided by the agent, N_o is other or non-sales related own effort provided by the agent, N_a is the sum of non-selling effort provided by assistants or other individuals, and $N = N_o + N_a$ represents the total other or non-sales related effort expended on behalf of the agent from all sources. The parameter θ is an index of agent income producing ability or overall productivity; this index is positive, with larger values corresponding to greater ability or productivity.

We assume that all effort is productive in the sense that it generates expected income ($f_S > 0$ and $f_N > 0$). We also assume that combinations of both types of effort yield greater expected income than selling effort alone, which allows for (but does not require) the notion that both types of activities will ultimately affect the flow of sales income. Denoting the marginal rate of technical substitution between selling and non-selling effort as $MRTS_{S,N} = f_S / f_N$, this assumption can be restated as diminishing $MRTS_{S,N}$ along an expected income isoquant in effort input {*S*,*N*} space, or $(dMRTS/dS)_{dI=0} < 0$. This formulation allows for but does not require overlapping effects of non-selling efforts on sales income.[3] The assumption of diminishing $MRTS_{S,N}$ means that the income function $\theta f(S,N)$ is strictly quasiconcave, an intuitively appealing property that is both economically meaningful and useful when deriving results. Therefore, $\theta f(S,N)$ is a well-behaved neoclassical production function with all of the usual properties. It is worthwhile to note, however, that the specific form of this production function can vary from individual agent to individual agent, reflected in the overall productivity index θ that captures differences in skills affecting an agent's ability to coordinate own efforts and hired effort of others to generate income from various activities (differences across agents in terms of their opportunities for generating non-selling income are considered later).

The disutility or opportunity cost of total effort to the agent is $\beta c(S+N_o)$, where greater agent effort comes at greater cost, $c' > 0$, and the marginal cost of effort is increasing, $c'' > 0$. The parameter β is a positive parameter that can vary across types of agents, where higher values indicate greater marginal cost of given effort, or higher opportunity cost of own effort, whether expended for selling or non-selling activities.

For non-selling related activities, the agent can choose to supply N_o effort himself or can hire an assistant or contract office staff at a price of p per unit of N_a provided. The ability of the

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[3] For example, let $f^s(S)$ be expected sales income and $f^n(N)$ be expected non-sales income for the agent so that total expected income is $f = f^s + f^n$. It is easy to verify that diminishing or constant marginal productivity of effort in both income-generating processes with diminishing marginal productivity in at least one (i.e., $f^s_{SS} \le 0$ and $f^n_{NN} \le 0$ with either $f^s_{SS} < 0$ or $f^n_{NN} < 0$) yields diminishing $MRTS_{S,N}$.

agent to subcontract this type of effort hinges upon the agent's ability to monitor N_a at reasonable cost. We assume that N_a denotes effective assistance effort (that is, productive effort) so that the price of effective contracted effort, *p*, includes any monitoring cost incurred by the agent.

Since we are focusing on agent behavior, throughout the following we assume that the primary activity is to generate sales income so that the agent will always expend some selling effort in equilibrium ($S > 0$). The agent's objective is to choose the own effort mix $\{S, N_o\}$ and how much outside assistance $\{N_a\}$ to hire to maximize expected agent income net of effort cost

$$\max_{S, N_o, N_a} \theta f(S, N_o + N_a) - \beta c(S + N_o) - pN_a \tag{1}$$

subject to the boundary constraints $N_o \geq 0$ and $N_a \geq 0$.

The Kuhn-Tucker conditions for this problem are

$$\begin{aligned} &\theta f_S - \beta c' = 0 \\ &\theta f_N - \beta c' \leq 0; \;\; [\theta f_N - \beta c']N_o = 0; \;\; N_o \geq 0 \\ &\theta f_N - p \leq 0; \;\; [\theta f_N - p]N_a = 0; \;\; N_a \geq 0 \end{aligned} \tag{2}$$

These conditions can be used to ascertain when the agent will employ outside help to assist in non-sales related tasks. Denoting the optimal solution to problem (1) with asterisks, complementary slackness (second line in (2)) means that $N_o^* > 0$ only if $\theta f_N(S^*, N_o^* + N_a^*) = \beta c'(S^* + N_o^*)$ at the optimal solution. Similarly, the third line in (2) means that $N_a^* > 0$ only if $\theta f_N(S^*, N_o^* + N_a^*) = p$. Therefore, the agent will apply some of his own effort to non-sales activities and will hire help only if the marginal cost of agent effort equals the marginal cost of hiring assistance, or $c'(S^* + N_o^*) = p$. On the other hand, if $\theta f_N < c'$ in equilibrium then the second line in (2) requires $N_o^* = 0$. Assuming the agent is engaging in some non-sales activities, $N_a^* > 0$ must then be true in this case, from which the third condition in (2) requires $\theta f_N = p$. Similar comparisons reveal $\theta f_N < p$ implies $N_a^* = 0$ in which case $\theta f_N = \beta c'$ and $N_o^* > 0$. Pulling these results together, the agent relies solely on own effort for non-sales activities when his marginal cost of effort is less than the marginal cost of hiring and monitoring services from others; the agent relies solely on hired efforts for non-sales activities when the marginal cost of own effort is greater than the marginal cost of hiring and monitoring services from others:

$$\begin{aligned} &\beta c' < p \rightarrow N_o^* > 0, N_a^* = 0 \\ &\beta c' = p \rightarrow N_o^* \geq 0, N_a^* \geq 0 \\ &\beta c' > p \rightarrow N_o^* = 0, N_a^* > 0 \end{aligned} \tag{3}$$

Therefore, other things equal, agents with higher opportunity costs (greater β) tend to concentrate their own effort more narrowly in their primary selling activities, relying on assistance of others for any non-selling related activities undertaken. Further, as demonstrated in the appendix, agents with higher opportunity costs of effort also tend to expend less total effort; in addition, the higher opportunity cost effect by itself leads to lower expected income from all sources.

But we can conclude more from the above conditions. Note that agent's selling effort condition, the first line in (2), implies $\theta f_S = \beta c'$ so that (3) can be restated in terms of the income generating abilities from selling effort, θf_S, as

$$\begin{aligned} &\theta f_S < p \rightarrow N_o^* > 0, N_a^* = 0 \\ &\theta f_S = p \rightarrow N_o^* \geq 0, N_a^* \geq 0 \\ &\theta f_S > p \rightarrow N_o^* = 0, N_a^* > 0 \end{aligned} \tag{4}$$

Thus, the agent relies solely on own effort (hired effort) for non-sales activities when his marginal value of selling effort is less than (greater than) the marginal cost of hiring and monitoring services from others. Cast in this way, the result is intuitively appealing: *agents with greater overall ability (higher θ) have a greater marginal value of selling effort and are therefore more inclined to pay for effort from others for help in non-selling activities; agents with less ability (lower θ) have a lower marginal value of selling effort and are therefore more inclined to divert own selling effort to non-sales activities rather than paying the relatively higher price to rely on assistance from others.*

The conditions summarized in (3) and (4) depend on the relative values of $\beta c'$ or θf_S and *p* in equilibrium, and as such they also imply that a lower price of hiring outside assistance increases the likelihood that the agent will do so, thereby allowing the agent to focus his own efforts more narrowly in selling activities To the extent that larger firms enjoy economies of scale in monitoring support staff activities, an agent in a larger firm who can draw on support staff at lower cost is more likely to focus his own effort more narrowly on selling activities than in a smaller firm environment. *The difference in effort mix across different size firms also implies higher expected income for agents in larger firms*, as demonstrated in the appendix.

These relationships are also useful for understanding how effort allocation differs across agents with different expected incomes in equilibrium. We start with the following result, the proof of which appears in the appendix: Agents with greater productivity earn greater expected income in equilibrium ($dI^* / d\theta > 0$). At the same time, though, (4) implies that agents with greater productivity are also more likely to specialize own efforts in selling activities. Pulling these two results together yields the effort level effect: *agents who specialize their efforts earn higher total income than those who do not.* This is a result that can be tested empirically.

The intuition for this result is as follows. Because agents with greater incomes are also expending greater total own effort $(\partial S^* / \partial\theta + \partial N_o^* / \partial\theta > 0)$, they also have a greater marginal cost of own effort c' in equilibrium because this marginal cost is rising with greater total effort. When the agent's total effort is sufficiently high, the marginal cost of effort rises above the price of hiring help, making it more profitable to hire outside assistance for non-sales activities in order to focus own effort more narrowly on selling activities. Higher total effort, however, translates directly into higher expected income, which yields the relationship spelled out above. But we do know more. Note that high θ agents not only expend greater total own effort, they expend greater sales effort (because they shift non-sales effort into sales activities). Therefore, the model also predicts that agents who specialize their efforts earn higher *sales* income than those who do not. (This particular result, however, is not testable with the data available for this study.)

Finally, we consider the reasonable possibility that agents may differ by their ability to generate non-selling related real estate income. This may reflect the lack of opportunity or skills. Regardless, differences in agent ability to generate non-selling related income are easily captured by introducing into the income production function another parameter, γ, where higher values of γ indicate an agent with lower ability (or less opportunity) to generate non-selling income; $I = \theta f(S, N; \gamma)$ such that $f_\gamma(S, N; \gamma) < 0$ and $dMRTS_{S,N} / d\gamma > 0$ at each effort combination $\{S, N\}$. The appendix demonstrates the intuitively appealing result: *agents with fewer opportunities to generate non-sales income not only specialize their own effort more heavily in selling activities, they tend to generate lower total income as well.*

3. The Data

The data for this study are drawn from surveys of the National Association of REALTORS membership profile for the years 2005, 2007, and 2008. The reports across years are similar, with some variation in questions across years. The survey examines the earnings and expenses of the licensee as well as demographics, use of technology, and details of the type and size of office in which the licensee works. In each of the years, the data related to income, sales expense, and transactions pertain to the prior year (i.e., in the 2005 survey, it is 2004 income) while the other variables pertain to the current year (i.e., marital status for 2005 is for that year).

The 2005 survey has 86 questions and was sent using a random sampling technique to 60,000 members via regular mail and another 60,000 in a web-based survey. The total returned was 8,450. The 2007 eighty seven question survey was sent to 70,000 members via mail and 70,000 through a web-based survey. The response rate was similar to the 2005 survey, with 10,777 surveys returned. The 2008 survey was sent to 72,000 members using regular mail and another 89,400 via a web-based survey. The response rate was again similar with 9,997 surveys returned. We use all surveys that contain responses for our variables of interest. In several rare occasions, we had to delete an observation due to a data point that would not be possible, such as a member being 1,000 years old, etc. This process generates 4,270, 4,804, and 4,556 usable observations in 2005, 2007, and 2008, respectively.

Table 1 lists the main variables used in the study; the next section provides the rationale for including each in the empirical model. The average gross income ranges between $36,000 and $42,000 (Table 1 reports presents the natural log used in the empirical models). Members have generally worked at their current firm an average of just over 6 years in each of the surveys and report just over 11 years of experience. The average agent is working around 40 hours per week and is just over 50 years old with a household size between 2 and 3 people. Not surprisingly, the average numbers of deals per member changed quite dramatically over the sample period, ranging from over 27 in 2005 and only 14 in 2008. These changes mirror sales volume changes in the overall market. The sales staff in each office fluctuated from a low of 25 in 2005 to a high of 68 in 2007.

4. The Empirical Model and Results

This section estimates an hedonic income function, specifying log of income as a function of the usual determinants motivated by human capital theory. All of the models are estimated for the years 2005, 2007, and 2008—years that span the end of the housing boom and the early periods of the subsequent decline. The separate samples reveal the extent to which the various market phases affect the estimates, if at all. Tables 2 and 3 report the OLS estimates for versions of the model all of the sample years. The right hand side variables for the hedonic income models include controls for agent productivity, firm size effects, personal characteristics, and effort and effort mix. We consider each, in turn.

Agent productivity. *Experience* and its square are included to control for productivity differences across agents that stem from non-firm-specific skills or human capital accumulated through the passage of time experience in the profession. As shown in the appendix, in the context of the theoretical effort allocation model greater agent productivity leads to greater expected income. In the context of standard human capital theory, the coefficient on *Experience* is expected to be positive and the coefficient on the quadratic term is expected to be negative (Rosen 1976). In the context of the agent effort allocation theory, greater human capital acquired over a longer period of experience in the profession leads to greater overall productivity, which yields higher expected income, other things equal. Referring to the estimates reported in Tables 2 and 3, experience and its square have the expected signs and significance in all of the models for all years, regardless of how effort mix is measured. This result is consistent with earlier agent income studies (Benjamin et al. 2000, Johnson et al. 2007).

Years Current Firm measures the number of years the agent has been with the current firm. The coefficient on this variable is expected to be positive, reflecting the cumulative effects of agents' investment in firm-specific human capital over time. This variable, too, has a significant positive effect on earnings in all of the models and in all years.

College is a dummy variable indicating that the agent earned at least a four year college degree. The coefficients on this variable are all positive and significant. Once again, this is as expected if we expect that college education increases productivity.

Firm size. The *Offices* variable measures the number of offices in the agent's firm and *Sale Staff Office* measures the number of agents in the agent's office. Both are measures of firm size. While the size of the agent's office has a significant positive effect on earnings, overall firm size, in terms of number of offices, does not. The office size effect is new. In the context of the effort allocation theory, larger offices are likely those with more trained support staff available to supply assistance in non-sales related activities (at lower effective p). Holding agent effort and own effort mix constant, this is expected to increase agent earnings. The insignificant firm size effect is not surprising given the range of sales price and liquidity effects and implied returns-to-scale found in the literature to date (Hughes 1995, Jud and Winkler 1994, Sirmans et al. 1991, Turnbull and Dombrow 2007, Turnbull and Sirmans 1993). On the other hand, Turnbull and

Dombrow (2007) argue that firm size variables are really picking up office size effects in many of these other studies. Their argument is consistent with what we observe in this data.

Large Expense is a dummy variable indicating agents reporting annual real estate business expenses of $25,000 or greater. The coefficient estimates on this variable are positive and significant in all samples and models; agents with greater out-of-pocket expenses tend to be those earning higher incomes.

Personal characteristics. Agent characteristics include *Age* and its square, a dummy variable *Male* for sex, a dummy variable *Married* for marital status, and *Household Size* as a measure of family composition. The coefficient estimates on *Age* and its square indicate that older agents enjoy higher income (holding experience, etc., constant). Male agents tend to earn significantly more than females (except in 2007 for the model reported in Table 2); the literature yields mixed results for agent sex whether measuring outcomes (selling price and liquidity) or agent income. While married agents earn significantly more than their unmarried counterparts, household size does not seem to matter in any of the models or samples.

Effort and activity mix. The total effort expended by the agent is measured by the *Hours Worked* variable and the *Part Time* dummy variable. The coefficients on these control variables are as expected—significantly positive and negative, respectively. These results indicate that greater effort yields greater total income—not surprising—and that part time agents earn significantly less (on an hourly basis) than full time agents, once differences in experience, etc., are taken into account.

We use two different measures of effort mix in the models. The model in Table 2 uses *Revenue Transactions*, the number of sales-related transactions during the year, and its square, as an effort mix control. The coefficients on these variables show the marginal effect on earnings of allocating more effort to selling related activities, holding total effort (time spent working) constant. The coefficient on *Revenue Transactions* is significant and positive in all cases, indicating that agents who allocate relatively more of their time to selling related activities enjoy higher incomes. The quadratic term coefficient is positive and significant, but extremely small. At the least, the estimate indicates that the percent return to additional total effort does not diminish with greater application of effort; it increases, however slightly.

The model reported in Table 3 uses a set of dummy variables to capture effort mix effects on income. First, we define the agent's main function as either listing or selling. The dummy variables *Lowest Main Function*, *Low Main Function*, *Medium Main Function*, *High Main Function*, and *Highest Main Function* indicate mixes of effort (or time) that are more and more concentrated on selling activities. *High Main Function* is the omitted category in the models reported in Table 3. All of the coefficients are negative, indicating that the highest income category is the omitted *High Main Function.* Looking at the estimates for this model using the 2005 data, the lowest main function yields a fraction of the income of the highest income category, but the income rises steadily as the effort mix tends more to selling related activities. The most concentrated selling effort mix, however, tends to significantly reduce total agent income. While the significance disappears for the medium main function dummy variable coefficient in 2007, the broad pattern remains the same across all years. The implication that increasing specialization increases expected income is consistent with the effort allocation theory and the results found in the other empirical models using *Revenue Transactions.* The decline in expected income for the most specialized agents, however, presents a major departure from those results.

To get a better feel for the magnitude of the income differences across the effort mix categories, Figure 1 compares the percentage income differential between agents in the high main function category (high degree of specialization in sales related activities) relative to those in the other effort mix categories for the 2005, 2007, and 2008 samples. The top panel shows the differences for each category by year; the bottom panel shows the differences for each category as an average over all three years. Overall, these estimates indicate that the agent maximizes income in the high main function category, experiencing a 55% and 20% higher income relative to agents in the lowest and low main function categories, respectively. As pointed out earlier, further specialization leads to lower income; agent income in the highest main function category (most specialized) is 10% lower than that for agents in the high function category.

The positive relationship between specialization in selling related activities and income is what we expect, given our effort allocation theory. But what explains the decline in income for the most specialized group of agents in the model reported in Table 3? Recall that greater agent opportunity cost (higher β in the effort mix model) increases the likelihood that the agent will concentrate own effort more in selling related functions, relying more heavily on assistance from others to perform non-selling related activities. We do not have good empirical controls for agent opportunity cost, so it is possible that the most specialized effort mix category includes many agents with higher opportunity cost and the coefficient is picking up this effect in the data. On the other hand, there may be a segment of the agent population that simply does not have the opportunity or skills to earn income from non-selling related activities (higher γ in the effort mix model). As shown earlier, these agents specialize more in selling related activities but also earn less. With the available data, however, we cannot empirically probe more deeply into the issue to see which, if either, rationale applies. At this point, we only can offer these cases as feasible explanations for the somewhat lower income observed for the most specialized agents when compared with their slightly less specialized (but still highly specialized) counterparts.

5. Conclusion

This is the first paper to examine the relationship between earnings and how residential real estate agents allocate their time. The effort-allocation theory offered here yields several testable results. Perhaps most importantly, the model illustrates that agents that are strong at their core income producing function have a high opportunity cost of doing anything except that function. These agents would be inclined to expend much of their effort on this function – in other words specialize. These agents are then shown to have higher levels of expected income. Additionally, the model shows agents with lower opportunity costs of time or fewer opportunities to earn income in ancillary activities also specialize. The opens up the possibility that there may be highly specialized agents that are not the highest income earners.

Our empirical results largely support the theoretical model in that, in general, higher effort and specialization is positively related to income. Additionally, we find that at the highest level of specialization income falls somewhat. As noted above, this result may reflect the mix of agent types in the population of most specialized agents. Additional data with more detailed individual agent information is needed to probe more deeply into this particular result.

Appendix

This appendix proves the following technical results claimed in the text. Define

$$\{S^*, N_o^*, N_a^*\} = \arg\max \theta f(S, N_o + N_a) - \beta c(S + N_o) - pN_a \text{ s.t. } N_o \geq 0; N_a \geq 0$$

which satisfy the Kuhn-Tucker conditions

$$\begin{aligned} &\theta f_S - \beta c' = 0 \\ &\theta f_N - \beta c' \leq 0; \; [f_N - \beta c']N_o = 0; \; N_o \geq 0 \\ &\theta f_N - p \leq 0; \; [f_N - p]N_a^* = 0; \; N_a \geq 0 \end{aligned}$$

Claim 1. $dI^* / d\beta < 0$, $dI^* / d\theta > 0$, *and* $dI^* / dp \leq 0$ *as* $N_a^* \geq 0$. ·

Proof. Substitute the agent's optimal solutions $\{S^*, N_o^*, N_a^*\}$ defined above into the objective function to derive the agent maximum expected income as a function of the parameters $\{\theta, \beta, p\}$

$$\begin{aligned} I^* &= \theta f(S^*, N_o^* + N_a^*) - c(S^* + N_o^*) - pN_a^* \\ &= I(\theta, \beta, p) \end{aligned}$$

Since this is an optimized function, apply the envelope theorem when differentiating with respect to each of the parameters to find

$$\begin{aligned} &\frac{dI^*}{d\beta} = -c(S^* + N_o^*) < 0 \\ &\frac{dI^*}{d\theta} = \frac{\partial I^*}{\partial \theta} = f(S^*, N_o^*) > 0 \\ &\frac{dI^*}{dp} = \frac{\partial I^*}{\partial p} = -N_a^* \leq 0 \text{ as } N_a^* \geq 0 \end{aligned}$$

The first part of the claim, $dI^* / dp \leq 0$ as $N_a^* \geq 0$, yields the prediction that agents in larger firms enjoying lower p earn higher expected incomes than otherwise identical agents in smaller firms without the advantage of lower cost non-sales effort from assistants.

Claim 2. $\partial S^* / \partial\theta + \partial N_o^* / \partial\theta > 0$.

Proof. There are three cases to consider: (i) $N_o^* > 0, N_a^* = 0;$ (ii) $N_o^* = 0, N_a^* < 0;$ and (ii) $N_o^* > 0, N_a^* > 0$. For (i) the Kuhn-Tucker conditions simplify to

$$\theta f_S - \beta c' = 0$$
$$\theta f_N - \beta c' = 0$$

Totally differentiate this system of equations and solve for the comparative static results in the usual way to find

$$\frac{\partial S^*}{\partial\theta} = \frac{f_N(\theta f_{SN} - \beta c'') - f_S(\theta f_{NN} - \beta c'')}{(\theta f_{SS} - \beta c'')(\theta f_{NN} - \beta c'') - (\theta f_{SN} - \beta c'')^2}$$
$$\frac{\partial N_o^*}{\partial\theta} = \frac{f_S(\theta f_{SN} - \beta c'') - f_N(\theta f_{SS} - \beta c'')}{(\theta f_{SS} - \beta c'')(\theta f_{NN} - \beta c'') - (\theta f_{SN} - \beta c'')^2}$$

The second order conditions for the maximum in this case are $(\theta f_{SS} - \beta c'') < 0$, ($\theta f_{NN} - \beta c'') < 0$, $D = (\theta f_{SS} - \beta c'')(\theta f_{NN} - \beta c'') - (\theta f_{SN} - \beta c'')^2 > 0$ so that the denominator of the above expression is positive. Summing the two derivatives above and substituting $\beta c' = \theta f_S = \theta f_N$ to simplify we get

$$\frac{\partial S^*}{\partial \theta} + \frac{\partial N_o^*}{\partial \theta} = \frac{(f_N + f_N)(\theta f_{SN} - \beta c'') - f_S(\theta f_{NN} - \beta c'') - f_N(\theta f_{SS} - \beta c'')}{D}$$

$$= \left(\frac{1}{\beta c'}\right)\frac{2\theta f_S f_N(\theta f_{SN} - \beta c'') - \theta f_S^2(\theta f_{NN} - \beta c'') - \theta f_N^2(\theta f_{SS} - \beta c'')}{D}$$

$$= \left(\frac{\theta^2}{\beta c'}\right)\left(\frac{2 f_S f_N f_{SN} - f_S^2 f_{NN} - f_N^2 f_{SS}}{D}\right) > 0$$

where $2 f_S f_N f_{SN} - f_S^2 f_{NN} - f_N^2 f_{SS} > 0$ can be recognized as the determinant of the bordered Hessian matrix for $f(S, N)$, which is positive for all strictly quasiconcave functions (i.e., exhibiting the neoclassical production function property of diminishing $MRTS_{SN}$, as in this case). This yields the result to be shown for case (i).

For (ii) the Kuhn-Tucker conditions simplify to

$$\theta f_S - \beta c' = 0$$
$$\theta f_N - p = 0$$

Totally differentiate this system and solve for the comparative statics in the usual way to find

$$\frac{\partial S^*}{\partial \theta} = \frac{f_N \theta f_{SN} - f_S \theta f_{NN}}{(\theta f_{SS} - \beta c'')\theta f_{NN} - \theta^2 f_{SN}^2}$$

The second order conditions for the maximum in this case are $(\theta f_{SS} - \beta c'') < 0$, $\theta f_{NN} < 0$, and $(\theta f_{SS} - \beta c'')\theta f_{NN} - \theta^2 f_{SN}^2 > 0$ so that the denominator of the above expression is positive. The numerator takes the sign of

$$\frac{f_S}{f_S} - \frac{f_{NN}}{f_N}$$

The first term is the percentage change in the marginal productivity of *S* as more *N* is applied to the income production process. The second term is the percentage change in the marginal productivity of N as more N is applied to the income production process. $f_{NN} < 0$ from the second order conditions the optimization problem for (ii). Therefore, if $f_{SN} \geq 0$ (i.e. non-selling effort and selling effort are unrelated or complementary inputs in the production of income) then the above expression is positive and $\partial S^* / \partial \theta > 0$. If $f_{SN} < 0$ (i.e., non-selling effort and selling effort are substitute inputs in the production of income) then the absolute value of the first ratio is less than the absolute value of the second ratio in the above expression when N is a better substitute for itself than S in the income production process. This means that S is not an inferior input in the production of income, a standard assumption for the neoclassical production function. Thus, when selling effort is a normal input in this sense, $\partial S^* / \partial \theta > 0$ in case (ii), which yields the result to be shown because $N_o^* = 0$.

Finally, for (iii), the Kuhn-Tucker conditions simplify to

$$\begin{aligned} \theta f_S - \beta c' &= 0 \\ \theta f_N - \beta c' &= 0 \\ \theta f_N - p &= 0 \end{aligned}$$

which yields a unique solution $\{S^*, N^*\}$ but a nonunique solution $\{S^*, N_o^*, N_a^*\}$ because any combination of own and hired non-sales efforts satisfying $N_o^* + N_a^* = N^*$ also satisfies the maximization problem. Therefore, by the implicit function theorem, the solution $\{S^*, N^*\}$ is differentiable in θ, p while $\{S^*, N_o^*, N_a^*\}$ is not. For decreases in θ conditions (i) hold and for increases in θ conditions (ii) hold, along with their derivative properties.

Claim 3. $dI^* / d\gamma < 0$.

Proof. Following the procedure above, the agent's maximum expected income is

$$I^* = \theta f(S^*, N_o^* + N_a^*; \gamma) - \beta c(S^* + N_o^*) - pN_a^*$$

Applying the envelope theorem yields

$$\frac{dI^*}{d\gamma} = \theta f_\gamma (S^*, N_o^* + N_a^*; \gamma) < 0$$

which is the result to be shown.

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Figure 1.A. Income Differential between Agents in High Main Function and Agents in other Functions during 2005, 2007, and 2008.

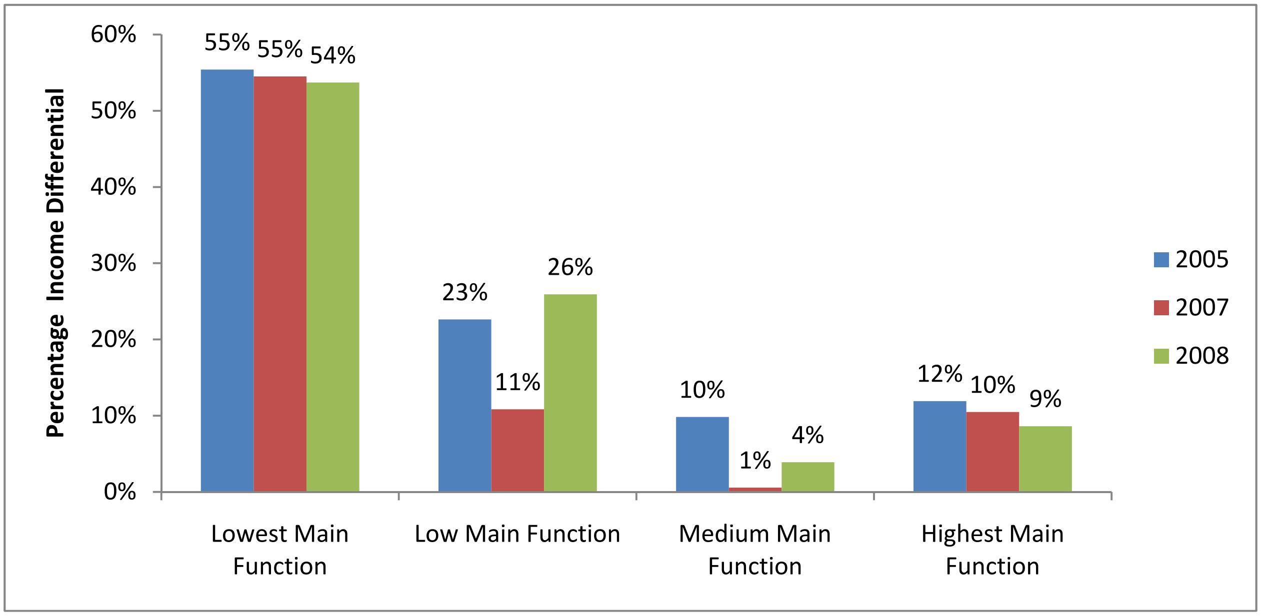

Figure 1.B. Average Income Differential between Agents in High Main Function and the Agents in the other Functions during 2005, 2007, and 2008.

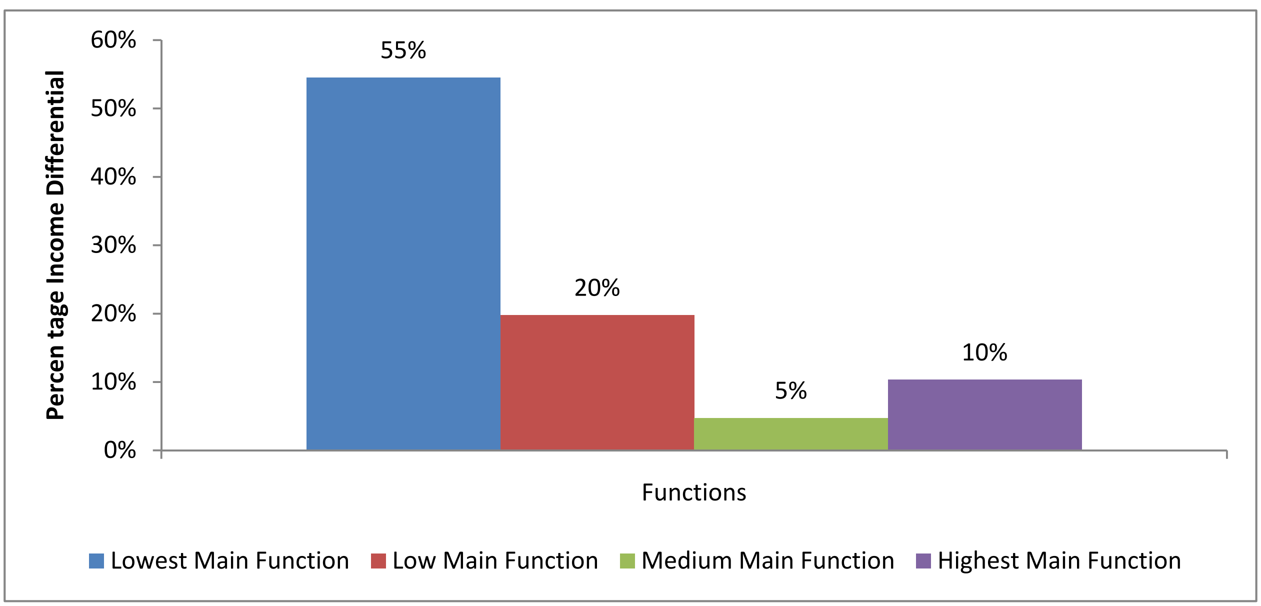

Table 1. Summary Statistics of the Gross Income in years 2005, 2007, 2008.

	Mean			Median			SD		
Year	2005	2007	2008	2005	2007	2008	2005	2007	2008
Ln(Gross Income)	10.66	10.64	10.51	10.66	10.66	10.66	1.24	1.21	1.25
Years Current Firm	6.38	6.39	6.69	4.00	4.00	4.00	7.07	7.02	7.15
Experience	11.67	11.12	11.78	8.00	7.00	8.00	10.71	10.11	10.38
Hours Worked	42.38	40.34	39.08	40.00	40.00	40.00	16.07	15.90	16.38
Sale Staff Office	25.95	67.72	56.69	25.00	15.50	15.50	18.83	174.35	124.29
Age	50.52	50.33	51.34	51.00	51.00	52.00	11.87	12.60	12.15
Household Size	2.53	2.69	2.58	2.00	2.00	2.00	1.15	2.11	1.44
Revenue Transactions	27.90	16.73	14.31	12.00	9.00	7.00	281.58	35.67	31.67
Offices	72.08	50.94	56.39	2.00	2.00	2.00	536.07	412.74	424.14

Year	2005	2007	2008
Part Time	33.70%	37.84%	40.68%
Full Time	66.30	62.16	59.32
Male	42.58	39.77	37.73
Female	57.42	60.23	62.27
Large Expense	26.47	25.45	24.08
Married	72.34	73.26	72.30
Single	27.66	26.74	27.70
College	45.64	44.28	46.15
Lowest Main Function	3.91	4.81	5.72
Low Main Function	4.14	3.65	3.92
Medium Main Function	9.99	10.84	9.65
High Main Function	43.78	41.20	39.52
Highest Main Function	38.18	39.50	41.19

Table 2. Hedonic Agent Income Model Estimates (Model I)

Year	**2005**	**2007**	**2008**
Constant	8.27529	8.77101	8.47269
	(0.00000)	(0.00000)	(0.00000)
Years Current Firm	0.00727	0.00611	0.00896
	(0.00493)	(0.01012)	(0.00027)
Experience	0.09503	0.07388	0.07615
	(0.00000)	(0.00000)	(0.00000)
Experience Squared	-0.00185	-0.00144	-0.00141
	(0.00000)	(0.00000)	(0.00000)
Hours Worked	0.01759	0.01570	0.01770
	(0.00000)	(0.00000)	(0.00000)
Part Time	-0.09966	-0.09257	-0.09062
	(0.03115)	(0.02907)	(0.03684)
Male	0.05216	0.04028	0.08055
	(0.06213)	(0.12003)	(0.00272)
Large Expense	0.84893	0.67688	0.70411
	(0.00000)	(0.00000)	(0.00000)
Sale Staff Office	0.00327	0.00008	0.00023
	(0.00002)	(0.25880)	(0.01915)
Age	0.02550	0.01931	0.02149
	(0.00048)	(0.01140)	(0.00633)
Age Squared	-0.00033	-0.00027	-0.00032
	(0.00000)	(0.00055)	(0.00007)
Married	0.09489	0.06893	0.11037
	(0.00721)	(0.02091)	(0.00087)
Household Size	0.02004	0.00979	0.00017
	(0.15716)	(0.13688)	(0.98898)
College	0.09592	0.07247	0.12365
	(0.00052)	(0.00427)	(0.00000)
Revenue Transactions	0.00469	0.01371	0.01929
	(0.00000)	(0.00000)	(0.00000)
Revenue Transactions Squared	-0.0000005	-0.00002	-0.00004
	(0.00000)	(0.00000)	(0.00000)
Offices	-0.00003	0.00008	-0.00001
	(0.19698)	(0.02908)	(0.67466)
Adjusted R^2	0.48561	0.45089	0.50063
Number of Observations	4270	4804	4556

Note: The numbers in parenthesis are p-values.

Table 3. Hedonic Agent Income Model Estimates (Model II)

Year	**2005**	**2007**	**2008**
Constant	8.55154	8.74815	8.72742
	(0.00000)	(0.00000)	(0.00000)
Years Current Firm	0.00778	0.00840	0.01211
	(0.00212)	(0.00018)	(0.00000)
Experience	0.09597	0.08505	0.08199
	(0.00000)	(0.00000)	(0.00000)
Experience Squared	-0.00182	-0.00166	-0.00149
	(0.00000)	(0.00000)	(0.00000)
Hours Worked	0.01687	0.01728	0.01826
	(0.00000)	(0.00000)	(0.00000)
Part Time	-0.12100	-0.10620	-0.10780
	(0.00776)	(0.00751)	(0.00882)
Male	0.05331	0.07256	0.09044
	(0.05414)	(0.00304)	(0.00041)
Large Expense	0.90364	0.82815	0.88639
	(0.00000)	(0.00000)	(0.00000)
Sale Staff Office	0.00354	0.00012	0.00022
	(0.00000)	(0.06962)	(0.02316)
Age	0.02258	0.02716	0.02218
	(0.00137)	(0.00015)	(0.00243)
Age Squared	-0.00031	-0.00038	-0.00035
	(0.00001)	(0.00000)	(0.00000)
Married	0.12174	0.06103	0.12633
	(0.00043)	(0.02904)	(0.00005)
Household Size	0.01272	0.00644	-0.00280
	(0.36246)	(0.27874)	(0.81316)
College	0.09090	0.08814	0.10469
	(0.00082)	(0.00022)	(0.00002)
Offices	-0.00004	0.00001	-0.00002
	(0.12766)	(0.69316)	(0.41216)
Lowest Main Function	-0.78018	-0.76681	-0.78938
	(0.00000)	(0.00000)	(0.00000)
Low Main Function	-0.21321	-0.14734	-0.30615
	(0.00347)	(0.02050)	(0.00000)
Medium Main Function	-0.10143	-0.00763	-0.05421
	(0.03295)	(0.84808)	(0.21177)
Highest Main Function	-0.13004	-0.10277	-0.07974
	(0.00001)	(0.00010)	(0.00350)
Adjusted R^2	0.477	0.442	0.473

Number of Observations	4481	5714	5522

Note: The numbers in parenthesis are p-values.